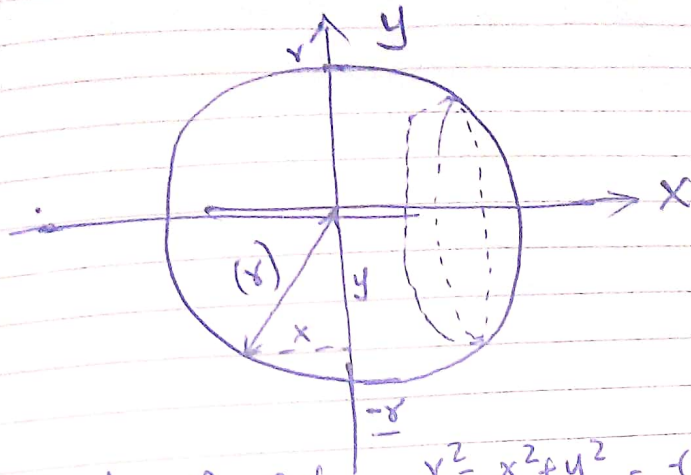


Question 1

(b)



Equation of circle $r^2 = x^2 + y^2$ - (1)

considering 1 disk = $\pi y^2 dx$

$$V = \lim_{n \rightarrow \infty} \sum_{i=1}^n \pi (y_i)^2 \Delta x = \int_{-r}^r \pi y^2 dx$$

Making y^2 subject $y^2 = r^2 - x^2$

$$V = 2 \int_0^r \pi (r^2 - x^2) dx = 2\pi \int_0^r (r^2 - x^2) dx$$

$$V = 2\pi \int_0^r (r^2 - x^2) dx = 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

$$V = 2\pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(r^2(0) - \frac{0^3}{3} \right) \right]$$

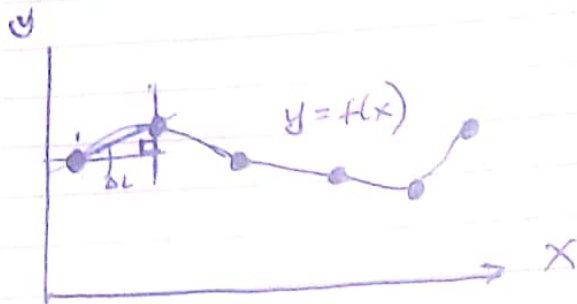
$$= 2\pi \cdot \frac{2}{3} r^3 = \frac{4}{3} \pi r^3$$

This will give the volume of the sphere.

Question 2

length of the curve.

Consider



using distance formula to compute d b/w two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

by Pythagorean theorem

$$\Delta L = \Delta x^2 + \Delta y^2$$

$$\Delta L = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Rightarrow \left(\sqrt{1 + \frac{\Delta y^2}{\Delta x^2}} \right) \Delta x \Rightarrow \Delta x = \sqrt{1 + \left(\frac{\Delta y}{\Delta x} \right)^2} \Delta x$$

ΔL (length of one segment)

$$= \Delta L = \sqrt{1 + \left(\frac{\Delta y}{\Delta x} \right)^2} \Delta x$$

As $\Delta x \rightarrow dx$
 $\Delta y \rightarrow dy$

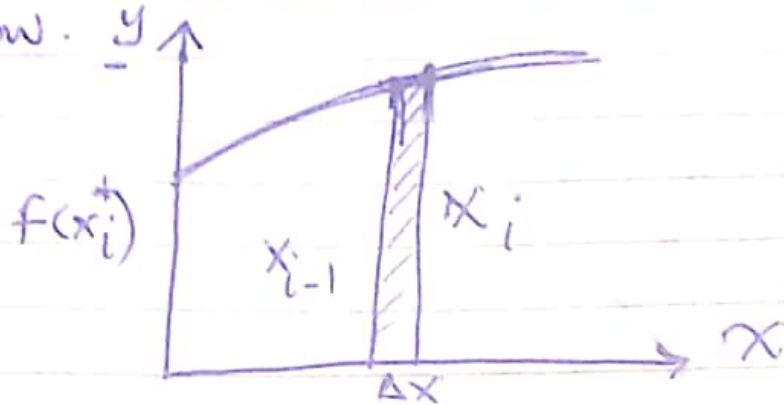
$$dL = \sqrt{1 + (f'(x))^2} dx$$

here taking a certain interval $a \rightarrow b$

Total length = sum of lengths of these segments

$$\therefore L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

Consider the strip on the axis below.



x_i - outer radius

x_{i-1} - inner radius

Vol of shell = Cross-section area $\times$ height of cylinder

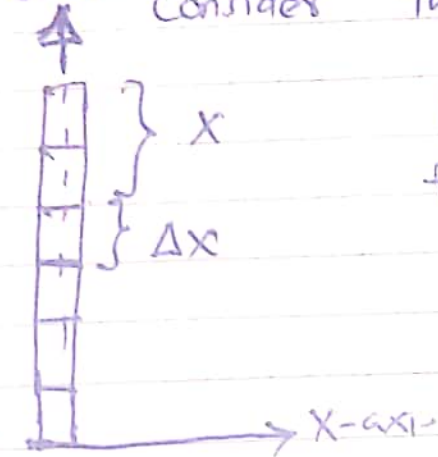
$$\begin{aligned} \text{Cross-section Area} &= \pi x_i^2 - \pi x_{i-1}^2 \\ &= \pi (x_i^2 - x_{i-1}^2) \end{aligned}$$

$$\begin{aligned} \text{Vol} &= f(x_i^*) \cdot \pi (x_i^2 - x_{i-1}^2) \\ &= \pi f(x_i^*) (x_i + x_{i-1}) (x_i - x_{i-1}) \\ &= 2\pi f(x_i^*) \left(\frac{x_i + x_{i-1}}{2} \right) (x_i - x_{i-1}) \end{aligned}$$

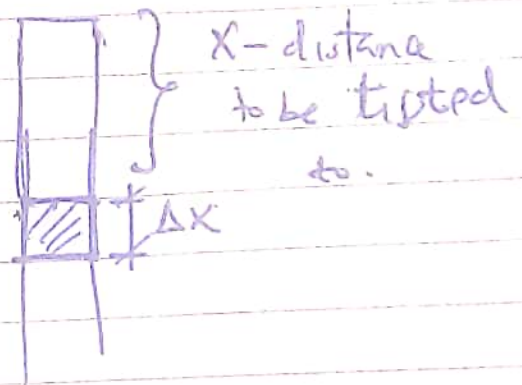
$$V_{\text{shell washer}} = 2\pi f(x_i^*) (\Delta x)$$

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Consider the diagram below.



$$W = F \cdot d$$
$$F = Ma$$



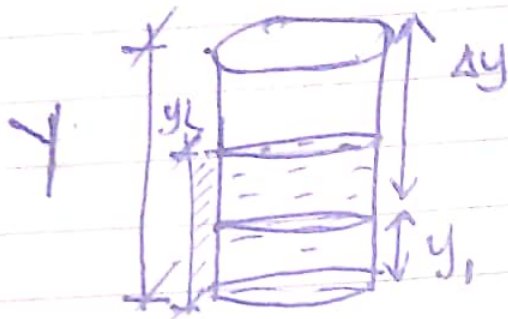
Force (F) required = ~~m~~ $\Delta x = M \cdot \Delta x$ m - Mass
distance of slice = X

$$W = F \cdot d$$
$$= \int M \cdot X \cdot \Delta x \quad (\text{one slice}).$$

Several slices:

$$W = \lim_{n \rightarrow \infty} \sum_{i=1}^n M X \cdot \Delta x$$

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water out of tank



$$F = (\pi r^2 h) \cdot M$$

↳ slice volume. M - mass

$$F = \pi r^2 \cdot \Delta y \cdot M$$

y_L - water
level

$$\text{distance} = Y - y_1$$

$$W = \pi r^2 (Y - y_1) \Delta y \quad \text{— one slice}$$

$$W = \pi r^2 \int_0^{y_L} (Y - y_1) dy$$